Procrastination on Long-Term Projects

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August 19, 2005

Abstract

We investigate naive procrastination on projects with multiple stages. In addition to classic procrastination in starting projects, naive people might undertake costly effort to begin projects but then never finish. Procrastination is more likely when the costs of completing different stages are more unequal, and it is when later stages are more costly that people start but don’t finish projects. Moreover, if the cost structure is endogenous, people are prone to choose cost structures that lead them to start but not finish projects. Some extensions further illustrate how people may incur costs on projects they never complete.

JEL Category: A12, D11, D91.

Keywords: Hyperbolic Discounting, Naivete, Present-Biased Preferences, Self Control, Time Inconsistency.

Acknowledgments: For helpful comments, we thank J. Barkley Rosser, Jr., an anonymous referee, and seminar participants at Wisconsin, Cornell, Johns Hopkins, Yale, Northwestern, Rice, Penn, the 2001 Winter Meetings of the Econometric Society, the 2001 CEPR/Ecares Conference on Psychology and Economics, and the 2001 SITE Workshop on Behavioral Economics. We thank Mandar Oak for valuable research assistance. For financial support, we thank the National Science Foundation (Awards SBR-9709485, SES-0078796, and SES-0079266), and Rabin thanks the Russell Sage and MacArthur Foundations.

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1. Introduction

People’s lives are filled with tasks that are onerous to perform, but which yield future benefits. Examples that will be salient to most readers are academic projects such as writing a dissertation, working on referee reports, or carrying out research, and also projects around the home such as shoveling the sidewalk, doing dishes, or cleaning the garage. But onerous tasks are far more widespread, and are important for labor economics — making investments in human capital, carrying out a job search — and for behavior in organizations such as firms — getting workers to carry out the various projects required to make the firm operate.

Do people carry out such tasks efficiently? Standard economic models assume that, except when there are externalities, the answer is yes. But recent behavioral-economic research on self-control problems — conceived of as a time-inconsistent preference for immediate gratification (as in Laibson (1997) and O’Donoghue and Rabin (1999a)) — suggests the answer may be no. In particular, people with such self-control problems are prone to procrastinate — to delay completion relative to what would be efficient. Understanding such procrastination is important both for understanding people’s behavior given the incentives they face, and for understanding how economic incentives are or should be set.\footnote{There are of course many rational reasons for people to delay completion of projects, and we do not claim that a taste for immediate gratification is needed to explain delays. Rather, we take the human taste for immediate gratification as well-established, and our goal is to better understand when and how this taste leads to additional \textit{inefficient} delay.}

The small but growing literature on time-inconsistent procrastination assumes that a “project” requires only a single period of effort, and is completed once begun.\footnote{A notable exception is Fischer (1999), as we discuss in Section 2. Other papers on procrastination include Prelec (1989), Akerlof (1991), and O’Donoghue and Rabin (1999a, 1999b, 2001).} Most real-world projects, in contrast, take some duration to complete, involve effort costs that vary for different stages of the project, and can be abandoned after begun. In this paper, we investigate a model of long-term projects in order to understand how these features influence procrastination.

In Section 2, we develop and analyze a model of long-term projects with an exogenous cost structure. We assume for simplicity that a project has two stages, which we refer to as “starting” and “finishing” the project. There is an infinite number of periods in which the person can work on the project, and in each period the person can either complete the current stage or do nothing. The person receives benefits only after completing the entire project.

Within this environment, we analyze three types of agents. As a benchmark, we consider standard time-consistent agents. But we also consider two types of agents who have self-control
problems. *Sophisticates* are fully aware of their future self-control problems and therefore correctly predict future misbehavior, and *naifs* are fully unaware of their future self-control problems and therefore believe they will behave in the future exactly as they currently would like to behave in the future. A comparison of these two extreme types will reveal the role of naivete in procrastination on long-term projects.

In this environment, standard time-consistent agents immediately start and then finish the project if and only if the discounted benefits are larger than the discounted costs, and otherwise they never start the project. Except for a few minor details — e.g., they won’t start if they expect not to finish — people with self-control problems who are sophisticated complete the project in much the same circumstances: When their taste for immediate gratification is relatively small, the discounted benefits need only be a little larger than the discounted costs to guarantee that sophisticates complete the project. In contrast, people with self-control problems who are naive might procrastinate: naifs may persistently *plan* to work on the project in the near future, but perpetually put off this work. Our main focus in this paper is such naive procrastination.3

Our previous research (see in particular O’Donoghue and Rabin (1999a, 2001)) has demonstrated naive procrastination on one-stage projects, and moreover that naive procrastination is more likely (and potentially more costly) for more onerous tasks. In this paper, we build on this intuition to identify several novel results that arise in the context of long-term projects. First, in addition to the classic form of procrastination wherein people never engage a valuable project, for long-term projects people may start but never finish. As a result, the cost of procrastination is not just the foregone benefits from a valuable project, but also the wasted effort — that is, people incur effort costs without ever receiving any benefits. Indeed, we show that such wasted-effort costs can be substantial. Second, whether and how a person procrastinates depends crucially on the structure of costs over the course of the project. Because it is the highest-cost stage on which people are most prone to procrastinate, procrastination is most likely when costs are allocated unevenly across stages. Moreover, when the allocation is uneven, the order of costs is important. When a project is difficult to start but easy to finish, a procrastinator is prone not to start. When a project is easy to start but difficult to finish, in contrast, a procrastinator is prone to start but not finish — and therefore incur costs without ever getting any benefits.

Whereas Section 2 assumes an exogenous cost structure, in Section 3 we endogenize this structure. Specifically, we allow the person to choose an uneven allocation with a disproportionate share

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3 While our formal analysis restricts attention to complete naifs, our main conclusions about procrastination extend to people who are only partially naive — who are aware that they will have future self-control problems but underestimate their magnitude.
allocated to stage 1, or an uneven allocation with a disproportionate share allocated to stage 2, or anything in between including an even allocation. If, for instance, a person must put in a total of 12 hours of effort but cannot work for more than 8 hours on any given day, then she must (plan to) put in 4 to 8 hours of effort on each of two days in some combination that totals 12 hours. This environment generates our third novel result: Because the same preference for immediate gratification that leads a person to procrastinate also leads her to prefer deferring as much cost as possible to stage 2, the person is prone to choose a cost structure that maximizes the likelihood that she will start the project but not finish it.

Much of our core analysis emphasizes the possibility of incurring effort costs without ever receiving any benefits. In Section 4, we briefly describe two extensions of our model that further explore this theme. Finally, Section 5 concludes.

2. Model with Exogenous Cost Structure

Consider a long-term project that consists of two stages, where completing each stage is onerous in the sense that completing it requires that the person incur an immediate cost. In this section, we assume that the cost structure is exogenous, where the first stage requires cost $c > 0$ and the second stage requires cost $k > 0$. We endogenize this structure in Section 3. Completion of the project generates future benefits. Specifically, we assume that completion of stage 2 in period $\tau$ initiates a stream of benefits $v \geq 0$ in each period from $\tau + 1$ onward. Although our formal analysis focuses for simplicity on two-stage projects, and also assumes that the person must complete the entire project before she can reap any benefits, we discuss in Section 5 how our lessons generalize.

There is an infinite number of periods in which the person can work on the project, and in each period the person can take one of two actions: She can complete the current stage or do nothing. Hence, in any period before which the person has not yet completed anything, she can choose either to do nothing or to complete the first stage; and in any period before which she has completed the first stage, she can choose either to do nothing or to complete the project.

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4 The crucial feature of an environment that generates the potential for procrastination is that costs are immediate whereas benefits are delayed. Here, our formal assumption is that if the person completes stage 1 in period $a$ and stage 2 in period $b > a$, then her instantaneous utilities are $u_a = -c$, $u_b = -k$, $u_\tau = v$ for all $\tau \in \{b + 1, b + 2, \ldots\}$, and $u_\tau = 0$ otherwise.

5 Whereas our model assumes that a long-term project consists of discrete stages, Fischer (1999) assumes that a long-term project consists of perfectly divisible effort — e.g., a person can allocate her total effort across days in any way she wants. One justification for our approach is that a “stage” is a task that must be performed over the course of a single day. A second justification, as we illustrate in Section 3, is that there is a minimum and maximum work level for any given day.
Within this environment, we investigate the behavior of people who have a time-inconsistent preference for immediate gratification. Specifically, we assume that a person’s intertemporal preferences at time $t$ can be represented by the following utility function:

$$U^t(u_t, u_{t+1}, \ldots, u_T) \equiv u_t + \beta \sum_{\tau=t+1}^T \delta^{t-\tau} u_\tau.$$ 

This two-parameter model is a simple modification of the standard one-parameter, exponential-discounting model, where the parameter $\beta$ represents a time-inconsistent preference for immediate gratification. For $\beta = 1$, these preferences are time-consistent. But for $\beta < 1$, at any given moment the person has an extra bias for now over the future.

As a benchmark, we analyze the behavior of standard time-consistent agents (who have $\beta = 1$), whom we refer to as TCs. We also consider two extreme types of agents with a preference for immediate gratification (who have $\beta < 1$). **Sophisticates** are fully aware of their future self-control problems and therefore correctly predict future misbehavior, and **naifs** are fully unaware of their future self-control problems and therefore believe they will behave in the future exactly as they currently would like to behave in the future. A comparison of these two extreme types will reveal the role of naivete in procrastination on long-term projects.

Recently (O’Donoghue and Rabin (2001)), we have formulated an approach to the more realistic assumption of partial naivete wherein a person is aware that she will have future self-control problems but underestimates their magnitude. In that paper, we show that, for one-stage projects,
all it takes is a little naivete to generate severe procrastination, and moreover that many features of procrastination that hold for complete naifs also hold for partial naifs (but not sophisticates). While we do not formally analyze partial naifs in this paper, we discuss in Section 5 how the same conclusion holds in this environment.\textsuperscript{10}

We next define formal solution concepts for TCs, sophisticates, and naifs. Let $A \equiv \{0, 1\}$ be the set of actions available in each period, where $a = 0$ means “do nothing” and $a = 1$ means “complete the current stage”. Let $h^t \in \emptyset, 1, 2, \ldots, t - 1\}$ be a history in period $t$, where $h^t = \emptyset$ means the person has not completed stage 1 prior to period $t$, and $h^t = \tau \in \{1, 2, \ldots, t - 1\}$ means the person completed stage 1 in period $\tau$. A strategy is a function $s$ such that if the history in period $t$ is $h^t$, then strategy $s$ specifies action $s(h^t, t) \in \{0, 1\}$. In the usual game-theoretic sense, a strategy is a plan for what to do in all possible contingencies; but we shall use strategies to represent both a person’s true behavior and her beliefs about future behavior, which may differ for naifs.\textsuperscript{11}

Let $V^t(a_t; h^t, s, \beta)$ represent the person’s period-$t$ preferences over current actions given history $h^t$ and conditional on following strategy $s$ beginning in period $t + 1$. Then:

$$V^t(a_t; h^t, s, \beta) \equiv \begin{cases} 
-c + \beta \delta^d \left(-k + \delta_v \frac{1}{1-\delta}\right) & \text{if } h^t = \emptyset, a_t = 1, \text{ and } \\
\beta \delta^d \left[-c + \delta^d \left(-k + \delta_v \frac{1}{1-\delta}\right)\right] & \text{if } h^t = \emptyset, a_t = 0, \text{ and } \\
-k + \beta \delta_v \frac{1}{1-\delta} & \text{if } h^t = \tau \neq \emptyset, \text{ and } a_t = 1 \\
\beta \delta^d \left(-k + \delta_v \frac{1}{1-\delta}\right) & \text{if } h^t = \tau \neq \emptyset, a_t = 0, \text{ and } \\
& \text{min}\{x > 0|s(t, t + x) = 1\} \\
& \text{min}\{x > 0|s(t + d, t + d + x) = 1\} \\
& \text{min}\{x > 0|s(t, t + x) = 1\} \\
& \text{min}\{x > 0|s(\tau, t + x) = 1\}.
\end{cases}$$

The four cases in this equation correspond to four different possibilities of when, relative to period $t$, the person completes the two stages. In the first case, the person completes the first stage now and the second stage in the future. In the second case, the person completes both the

\textsuperscript{10}For a formal analysis of partial naifs in this environment, see O’Donoghue and Rabin (2002).

\textsuperscript{11}We define strategies to depend on $t$ because our notation for histories does not identify the current period. Hence, $s(\emptyset, t)$ refers to whether the person would start the project in period $t$ conditional on not having started it yet, and $s(\tau, t)$ refers to whether the person would finish the project in period $t$ conditional on having completed stage 1 in period $\tau$. Also, our formulation rules out mixed strategies; it is perhaps best to interpret our analysis as applying to equilibrium strategies for an infinite horizon that correspond to some equilibrium strategy for a long, finite horizon, which (generically) does not involve mixed strategies.
first stage and the second stage in the future. In the third case, the person has completed the first stage in the past (in period $\tau < t$) and completes the second stage now. In the fourth case, the person has completed the first stage in the past and completes the second stage in the future.

With this notation, we can formally define our solution concept:

**Definition 1.** For all $\beta$, $\delta$, $c$, $k$, and $v$:

1. A strategy $s^{TC}$ is a **perception-perfect strategy for TCs** if, for all $t$ and $h^t$,
   $$s^{TC}(h^t, t) = \arg \max_{a \in \{0, 1\}} V^t(a; h^t, s^{TC}, 1).$$

2. A strategy $s^S$ is a **perception-perfect strategy for sophisticates** if, for all $t$ and $h^t$,
   $$s^S(h^t, t) = \arg \max_{a \in \{0, 1\}} V^t(a; h^t, s^S, \beta).$$

3. A strategy $s^N$ is a **perception-perfect strategy for naifs** if, for all $t$ and $h^t$,
   $$s^N(h^t, t) = \arg \max_{a \in \{0, 1\}} V^t(a; h^t, s^{TC}, \beta)$$
   where $s^{TC}$ is a perception-perfect strategy for TCs.

A perception-perfect strategy requires that in all situations a person choose optimally given her current preferences and her perceptions of her own future behavior. For TCs and sophisticates, who have correct perceptions about future behavior, a perception-perfect strategy merely satisfies the usual property that in all situations it specifies an optimal action conditional on following the strategy in the future. Because TCs have time-consistent preferences, any perception-perfect strategy will induce an optimal path of behavior given their current preferences. For sophisticates, in contrast, a perception-perfect strategy need not induce an optimal path of behavior. Unlike TCs and sophisticates, naifs have incorrect perceptions about their future behavior. In particular, they believe they will behave optimally in the future, which implies, given the structure of their preferences, that they believe that they will behave like TCs in the future. Hence, a perception-perfect strategy for naifs calls for, in all situations, an action that is optimal conditional on following a TC perception-perfect strategy in the future.

Lemma 1 describes the behavior of TCs:

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12 Throughout we assume for simplicity that when a person is indifferent between $a = 0$ and $a = 1$, she chooses $a = 1$. 
Lemma 1 (Behavior of TCs). For all $\delta$, $c$, $k$, and $v$:

(1) There exists a unique perception-perfect strategy $s^{TC}$ for TCs.

(2) The unique $s^{TC}$ must involve either (i) never starting the project or (ii) completing the project (immediately); specifically, $s^{TC}$ involves completing the project if and only if

$$-c + \delta \left[ -k + \frac{\delta v}{1 - \delta} \right] \geq 0.$$ 

Part 1 establishes that there is a unique perception-perfect strategy for TCs. Intuitively, in any situation, TCs follow an optimal path of behavior, and given our assumption that people complete the current stage when indifferent, there is a unique optimal path. Part 2 establishes that TCs either complete the project or never start the project, and which they do merely depends on whether the project is worth doing — in the sense that they prefer completing the project immediately to never completing the project. In other words, the condition for whether TCs complete the project is merely the standard net-present-value calculation applied to our environment. An important implication is that, holding constant the stage-1 net present value — holding constant $-c + \delta \left[ -k + \frac{\delta v}{1 - \delta} \right]$ — changing the distribution of costs over the course of the project does not affect the behavior of TCs.13

Indeed, changing the distribution of costs and benefits in a way that leaves the net present value unchanged cannot affect behavior for TCs, though it can for those with present-biased preferences.

Lemma 2 describes the behavior of sophisticates:

Lemma 2 (Behavior of Sophisticates). For all $\beta$, $\delta$, $c$, $k$, and $v$:

(1) There exists a perception-perfect strategy $s^{S}$ for sophisticates.

(2) Any $s^{S}$ must involve either (i) never starting the project or (ii) completing the project (eventually); specifically:

(a) If either $-c + \beta \delta \left[ -k + \frac{\delta v}{1 - \beta \delta} \right] < 0$ or $-k + \frac{\beta \delta v}{1 - \beta \delta} < 0$, then every $s^{S}$ must involve never starting the project;

(b) If $-k + \frac{\beta \delta v}{1 - \beta \delta} > 0$ and $-c + \beta \delta^{\gamma + 1} \left[ -k + \frac{\delta v}{1 - \beta \delta} \right] > 0$, where

$$\gamma \equiv \min \left\{ d \in \{0, 1, \ldots\} \mid -k + \frac{\beta \delta v}{1 - \beta \delta} \geq \beta \delta^{d+1} \left[ -k + \frac{\delta v}{1 - \beta \delta} \right] \right\},$$

then every $s^{S}$ must involve completing the project; and

(c) If $-k + \frac{\beta \delta v}{1 - \beta \delta} > 0$ and $-c + \beta \delta \left[ -k + \frac{\delta v}{1 - \beta \delta} \right] > 0 > -c + \beta \delta^{\gamma + 1} \left[ -k + \frac{\delta v}{1 - \beta \delta} \right]$, then there exist both some $s^{S}$ that involve completing the project and some $s^{S}$ that involve never starting the project.

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13 Indeed, changing the distribution of costs and benefits in a way that leaves the net present value unchanged cannot affect behavior for TCs, though it can for those with present-biased preferences.
Part 1 establishes that a perception-perfect strategy for sophisticates exists. But unlike TCs, sophisticates might have multiple perception-perfect strategies. As in O’Donoghue and Rabin (2001), the source of this multiplicity is an indeterminacy in when sophisticates complete a stage. To illustrate, consider a one-stage project, and suppose, for instance, that on any given day sophisticates prefer to complete the stage tomorrow rather than today, but also prefer today over two days from now — that is, they are willing to tolerate a one-day delay, but not a two-day delay. Then, given an infinite horizon, there will be one perception-perfect strategy under which sophisticates complete the stage on day 1, and another perception-perfect strategy under which sophisticates complete the stage on day 2.\footnote{Formally, the former strategy would specify completing the stage on days 1, 3, 5, ..., while the latter strategy would specify completing the stage on days 2, 4, 6, ...}

In this paper, we ignore finite delays in completing a task, and focus solely on how many stages people complete.\footnote{As discussed in some detail in O’Donoghue and Rabin (2001), there are ways of intuiting and formalizing how finite delays in completing a task (or, as here, the first stage of a task) are in this context qualitatively different and less important than the infinite delays we focus on.} On this dimension, Part 2 establishes that sophisticates behave much like TCs in that they either complete the project or never start the project. But part 2 also reflects that there are multiple reasons for sophisticates not to start.\footnote{Part 2 ignores the knife-edge cases in which one or more of the conditions hold with equality. These cases do not contradict our discussion in the text, but involve some technical subtleties that are irrelevant to our main conclusions.} First, like TCs, they might not start because the project is not worthwhile: when \(-c + \beta \delta \left[-k + \frac{\delta v}{1-\delta}\right] < 0\), they prefer never completing the project to completing the project immediately. But unlike TCs, sophisticates also might not start because they expect not to finish: when \(-k + \frac{\beta v}{1-\delta} < 0\), even if they started, they’ll prefer never finishing to finishing it immediately. Finally, even if the project is worthwhile and they expect to finish, sophisticates might not start if they expect too much delay before finishing. More precisely, if \(-c + \beta \delta \left[-k + \frac{\delta v}{1-\delta}\right] > 0\) and \(-k + \frac{\beta v}{1-\delta} > 0\), some perception-perfect strategies involve completing stage 2 immediately after completing stage 1, and all such strategies imply starting and finishing the project. But if their tolerance for delay of stage 2 — i.e., the \(\gamma\) in Lemma 2 — is greater than zero, then there are also some perception-perfect strategies that involve completing stage 2 \(\gamma + 1\) periods after completing stage 1. These perception-perfect strategies imply starting and (eventually) finishing the project if and only if \(-c + \beta \delta^{\gamma+1} \left[-k + \frac{\delta v}{1-\delta}\right] > 0\).
Even so, Proposition 1 describes a sense in which sophisticates behave much like TCs unless their preference for immediate gratification is large ($\beta$ far from 1).

**Proposition 1.** Define $C \equiv c + \delta k$ and $V \equiv \frac{\delta^2 v}{1-\delta}$, so that TCs never start if and only if $C/V > 1$. Then sophisticates never start only if $C/V > \beta$.

Proposition 1 establishes that if TCs complete the project while sophisticates don’t (for whatever reason), it must be that the ratio of costs to benefits is “close” to one, in the sense of being between $\beta$ and one. As our results below shall illustrate, procrastination can lead naifs to never start even as the ratio of costs to benefits approaches zero.

Finally, Lemma 3 describes the behavior of naifs:

**Lemma 3 (Behavior of Naifs).** For all $\beta$, $\delta$, $c$, $k$, and $v$:

1. There exists a unique perception-perfect strategy $s^N$ for naifs.
2. The unique $s^N$ must involve either (i) never starting the project, (ii) completing the project (immediately), or (iii) starting the project (immediately) but never finishing it; specifically:
   
   (a) $s^N$ involves completing stage 1 if and only if $-c + \beta \delta \left[-k + \frac{\delta v}{1-\delta}\right] \geq \beta \delta \left[-c - \delta k + \frac{\delta^2 v}{1-\delta}\right]$; and
   
   (b) $s^N$ involves completing stage 2 if and only if $-k + \frac{\beta \delta v}{1-\delta} \geq \beta \delta \left[-k + \frac{\delta v}{1-\delta}\right]$.

Part 1 establishes that, as for TCs, there is a unique perception-perfect strategy for naifs. The intuition is much the same: in any situation, naifs begin following an optimal path of behavior, and given our assumption that people complete the current stage when indifferent, there is always a unique optimal path. Part 2 establishes that, like TCs and sophisticates, naifs might never start the project or complete the project; however, because they revise their plans as time passes, naifs might also start the project but then never complete the project.

In fact, there are two reasons why naifs might not complete a stage. As for TCs and sophisticates, they won’t work on the project if it is not worthwhile. In addition, they might “procrastinate”: even when they want to (and expect to) complete the project, if they prefer to start working next period rather than now, then they will always plan to start working next period, and as a result they end up never working. The conditions in Part 2 of Lemma 3 reflect that the binding condition is whether they procrastinate.

17 Throughout the paper, we use this precise definition of procrastination. Although this definition makes it tautological that sophisticates cannot procrastinate, we feel there is a sense in which this use is correct, because Proposition 1 indicates that any delay by sophisticates is limited in severity to the influence that the taste for immediate gratification has on the overall cost-benefit evaluation.
In general, we doubt the importance of misbehavior driven by worthwhileness concerns; for instance, one could formalize a sense in which the degree of harm from any such misbehavior can be large only if $\beta$ is significantly less than 1. Hence, our main interest is in naive procrastination. In order to lay bare the forces that influence procrastination, we examine behavior when $\delta \to 1$, in which case everything is worthwhile, and therefore the only reason a person might not complete the project is procrastination. Proposition 2 characterizes how the different types behave when $\delta \to 1$.\footnote{Formally, for each statement, we prove that there exists $\tilde{\delta} < 1$ such that the statement holds for all $\delta \in (\tilde{\delta}, 1)$.}

**Proposition 2.** When $\delta \to 1$, for any $\beta$, $c$, $k$, and $v$:

1. TCs and sophisticates complete the project; and

2. Naifs complete stage 1 if and only if $c < \frac{\beta v}{1 - \beta}$, and if they complete stage 1, then they complete stage 2 if and only if $k < \frac{\beta v}{1 - \beta}$.

Part 1 captures the intuition that everything is worth doing when $\delta \to 1$, so both TCs and sophisticates complete the project. Part 2 captures that, even though the project is worthwhile, naifs might still not complete the project because they procrastinate. Indeed, Proposition 2 yields several novel conclusions about procrastination that arise in the context of long-term projects.\footnote{All of our qualitative conclusions when $\delta \to 1$ also hold when $\delta < 1$, but the equations become more complicated.}

First, two types of procrastination potentially arise when naifs face a long-term project. There is the classic form highlighted in previous procrastination papers wherein a person plans to start a valuable project but never does so. But for long-term projects, a person might instead start a valuable project planning to finish it, but never finish it. This novel form of procrastination is clearly worse, because the person incurs the cost associated with starting the project without ever accruing any benefits. In fact, Proposition 3 establishes that there is in principle no bound on how much effort a person might exert in starting a project she does not finish.

**Proposition 3.** For all $\beta$ and $\delta$, for any $c$ there exists $k$ and $v$ such that naifs complete stage 1 but never complete stage 2.

A second novel conclusion from Proposition 2 is that whether and how a person procrastinates depends crucially on the structure of costs over the course of the project. Naifs complete the project if and only if $\max\{c, k\} < \frac{\beta v}{1 - \beta}$. Hence, the allocation of costs over the course of the project...
becomes crucial. In particular, for any fixed total cost, naifs are most likely to complete the project when these costs are allocated evenly over the course of the project. If total costs are $\Gamma$, naifs are most likely to complete the project when $c = k = \Gamma/2$, and least likely to complete the project when $c = \Gamma$ or $k = \Gamma$. The intuition for the role of allocation is simple: Naifs complete the project if and only if they don’t procrastinate the highest-cost stage, and allocating costs evenly minimizes the cost of the highest-cost stage. Moreover, when costs are allocated unevenly, the order of costs is important, because it determines whether naifs incur costs without accruing benefits. In particular, again because it is the high-cost stage on which people are most prone to procrastinate, naifs incur costs without benefits only if the high-cost stage comes second — that is, only if $c < k$.

3. Model with Endogenous Cost Structure

The previous section shows how naive procrastination depends critically on the structure of costs over the course of a project. In this section, we endogenize this structure. To motivate our formal analysis, consider a person who must complete an unpleasant project at work that requires a total of 12 hours of effort. The person can put in these hours in any way that she wants, except that she cannot work for more than 8 hours on any given day, so that the project requires (at least) two days of work. But the person has discretion over how to allocate her time over those two days. What work schedule will the person choose?

Formally, we assume the person must choose a cost structure 

$$(c, k) \in \{(q, r) \mid q + r = A, q \leq \bar{a}, r \leq \bar{a}\} \equiv P(A, \bar{a}).$$

This formulation incorporates two assumptions. First, the total cost to be incurred, which we denote by $A$, is independent of the cost structure — that is, there are constant returns to effort within a period. We address at the end of this section how efficiency concerns might alter our conclusions. Second, there is a maximum cost $\bar{a}$ that can be incurred in any specific period. To make it meaningful that we are analyzing “long-term projects”, it must be that the person cannot choose to complete the project all at once, because otherwise TCs would choose to do so and naifs would plan to do so. We assume $A \in (\bar{a}, 2\bar{a})$, which makes it a two-period project.20

More generally, if $A$ and $\bar{a}$ were such that $A \in ((N - 1)\bar{a}, N\bar{a})$, then the project would be an $N$-period project. Also, our formal analysis restricts the person to incur costs in at most two periods. This restriction is irrelevant for TCs and naifs, who see no value in dividing up the costs more than necessary. But since sophisticates might see value to further dividing up the costs if doing so influences future behavior, this restriction might lead us to understate their likelihood of completing the project.

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Importantly, the person makes her choice of cost structure at the moment of action. Hence, if in period \( t \) the person has waited in all prior periods, then in period \( t \) she can choose either to wait again or to incur any cost \( q \in [A - \bar{a}, \bar{a}] \) while planning to incur cost \( A - q \) in some future period. If, in contrast, the person has incurred cost \( q \in [A - \bar{a}, \bar{a}] \) in the past, then in period \( t \) she can choose either to wait or to incur cost \( A - q \) to finish the project. Again, the person receives an infinite stream of benefits with per-period benefit \( v \geq 0 \) upon completion of the project.

Lemma 4 characterizes the behavior of the three types in this environment:

**Lemma 4.** For any \( \beta, \delta, v, A, \) and \( \bar{a} \) such that \( A \in (\bar{a}, 2\bar{a}) \):

1. **(TCs)** If \( -(A - \bar{a}) + \delta \left[ -\bar{a} + \frac{\delta v}{1-\delta} \right] \geq 0 \), then TCs complete project \((A - \bar{a}, \bar{a})\) immediately; otherwise they never start any project.

2. **(Naifs)** As long as \( -(A - \bar{a}) + \delta \left[ -\bar{a} + \frac{\delta v}{1-\delta} \right] \geq 0 \), naifs plan to complete project \((A - \bar{a}, \bar{a})\); in actuality, they complete stage 1 if and only if \( -(A - \bar{a}) + \beta \delta \left[ -\bar{a} + \frac{\delta v}{1-\delta} \right] \geq \beta \delta \left[ -(A - \bar{a}) - \delta \bar{a} + \frac{\delta v}{1-\delta} \right] \), and they complete stage 2 if and only if \( -\bar{a} + \frac{\beta \delta v}{1-\delta} \geq \beta \delta \left[ -\bar{a} + \frac{\delta v}{1-\delta} \right] \).

3. **(Sophisticates)** If \( -(A - r_o) + \beta \delta \left[ -r_o + \frac{\delta v}{1-\delta} \right] \geq 0 \), where \( r_o \equiv \min \left\{ \bar{a}, \frac{\beta \delta v}{1-\delta} \right\} \), then there exists an \( s^S \) under which sophisticates complete project \((A - r_o, r_o)\); otherwise they never start any project.

Lemma 4 captures the intuition that when the cost structure is endogenous, people will prefer to defer as much of the cost as possible to the second stage. Parts 1 and 2 establish that if TCs and naifs (plan to) work on any project, it involves deferring the maximum possible amount \( \bar{a} \) to the second stage. Part 3 establishes a similar result for sophisticates, but with a caveat: Because they expect to have future self-control problems, they will not allocate so much cost to stage 2 so as to make stage 2 not worthwhile, which requires that the stage-2 cost be no larger than \( \frac{\beta \delta v}{1-\delta} \). This propensity to defer costs is driven by both the person’s time-consistent impatience, as captured by \( \delta \), and the person’s preference for immediate gratification, as captured by \( \beta \).

In order to explore the implications of “endogenizing the cost structure,” we compare a person’s behavior given exogenous cost structure \((c, k)\) to her behavior given endogenous cost structure \( P(c + k, \bar{a}) \). We also assume \( c \leq \bar{a}, k \leq \bar{a}, \) and \( c + k > \bar{a} \), so that we are comparing behavior

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Lemma 4 does not provide a complete characterization of the set of perception-perfect strategies for sophisticates because such characterization is quite complicated. In addition to the details described in Section 2, here there is scope to construct additional equilibria based on different stage-2 behavior for different projects. Once again, we view such details as unimportant; and Proposition 4 establishes that they disappear when \( \delta \rightarrow 1 \).
under exogenous cost structure \((c, k)\) to behavior under endogenous cost structure \(P(c + k, \bar{a})\) when \((c, k) \in P(c + k, \bar{a})\).

Endogenizing the cost structure makes it more likely that the project is worth doing, because with discounting it is always better to allocate more of the total cost to the second stage. Hence, TCs are more likely to complete the project given endogenous cost structure \(P(c + k, \bar{a})\) than they are given exogenous cost structure \((c, k)\). In addition, endogenizing the cost structure also makes it more likely that sophisticates expect to want to finish — because they choose the cost structure accounting for stage-2 incentives. Hence, sophisticates are also more likely to complete the project given endogenous cost structure \(P(c + k, \bar{a})\) than they are given exogenous cost structure \((c, k)\).²²

Considerations of whether completing the project is worthwhile are relevant for naifs as well, and so for \(\delta < 1\) endogenizing the cost structure may make it more likely that they complete the project. But endogenizing the cost structure might also influence whether naifs procrastinate. In order to lay bare the implications of endogenizing the cost structure for procrastination, we again examine behavior when \(\delta \to 1\), in which case the above worthwhileness considerations disappear. Proposition 4 formalizes the effects of endogenizing the cost structure for naive procrastination.

**Proposition 4.** When \(\delta \to 1\), for any \(v, \bar{a}\), and \((c, k)\) with \(c, k \leq \bar{a}\) and \(c + k > \bar{a}\):

1. TCs and sophisticates complete the project both under exogenous cost structure \((c, k)\) and under endogenous cost structure \(P(c + k, \bar{a})\); and

2. Naifs start the project under endogenous cost structure \(P(c + k, \bar{a})\) if they start the project under exogenous cost structure \((c, k)\), and they complete the project under endogenous cost structure \(P(c + k, \bar{a})\) only if they complete the project under exogenous cost structure \((c, k)\).

Part 1 merely restates our earlier conclusion that when \(\delta \to 1\), everything is worth doing, and so TCs and sophisticates complete the project. Part 2 establishes that for naifs, endogenizing the cost structure makes it more likely that they start the project, while at the same time makes it less likely that they complete the project. The intuition is simple. We saw in Section 2 that naifs are prone to start but not finish when a disproportionate share of the total cost is allocated to stage 2. When the cost structure is endogenous, the same preference for immediate gratification that leads people to procrastinate also leads them to defer as much of the total cost as is possible to stage 2. Hence, they choose cost structures on which they are prone to start but not finish.

²² While the formal result for TCs is straightforward, the formal result for sophisticates is somewhat tricky given the existence of multiple perception-perfect strategies. But one could show, for instance, that it becomes more likely that every perception-perfect strategy involves completing the project, and that it becomes less likely that every perception-perfect strategy involves never starting the project.
Our results in Proposition 4 can be interpreted in terms of an intuition identified in O’Donoghue and Rabin (2001). One of the main results in that paper is that providing a person with additional options can make procrastination more likely. This outcome occurs when some new option is better than existing options from a long-run perspective but more onerous to carry out, because then the person will plan to do the new option, but may repeatedly put off incurring the high immediate cost. While that paper demonstrates that one can construct examples in which expanded choice exacerbates procrastination, one can also construct examples in which it mitigates procrastination. Here, by contrast, we show that a natural way to expand choice — giving people discretion over how to schedule their efforts — unambiguously makes naifs more prone to procrastinate.

Our results above are also suggestive of the implications of endogenizing the cost structure in a different way. Suppose that, rather than being able to allocate the total costs in a continuous fashion, a person must complete a project that consists of specific sub-component tasks, and can choose only the order of these tasks. Applying the logic above, because people prefer to defer costs to the second stage, naifs will plan to do the low-cost stage first and the high-cost stage second. Hence, endogenizing the order again makes it more likely that naifs incur costs without benefits.

We conclude this section by briefly discussing how our conclusions change when the choice of cost structure has efficiency implications. If there are increasing returns to effort within a period, then efficiency considerations militate in favor of an uneven allocation, since it is most efficient to try to do as much as possible on the longer day. This feature reinforces the propensity of naive procrastinators to defer as much cost as possible to the future, and hence reinforces our results. In contrast, if there are decreasing returns to effort within a period, efficiency considerations militate in favor of an even allocation. Hence, the person might not choose to defer as much cost as possible to stage 2, and as a result, the effects of endogenizing the cost structure become ambiguous.

4. Incurring Costs Without Benefits

Our basic model illustrates the possibility that naive procrastination may take the form of incurring effort costs without ever receiving any benefits. In this section, we explore two extensions of our model that expand on this theme.

Our first extension examines a natural class of long-term projects in which a person first carries out “research” and then later carries out “development”. Moreover, the duration of the research phase is endogenous because the person must choose when she has discovered something worth developing. Consider, for example, a Ph.D. student writing her dissertation. She starts by con-
ducting some research — she pursues a new idea and derives some preliminary results. Upon completion of this research, the student, with guidance from her advisor, assesses the likely quality of a dissertation based on these preliminary results. The student must then decide whether to develop these results into her final dissertation, or to return to conducting research on some new idea.

We can capture such situations with a simple extension of our model. Suppose that, in any period, a person can either (i) conduct research, which requires an immediate cost \( c > 0 \), (ii) develop an idea generated by past research, which requires an immediate cost \( k > 0 \), or (iii) do nothing. Each time the person conducts research, she receives a draw \( v \) from some distribution \( F(v) \), which reflects the per-period flow benefit that she would receive if she were to develop that idea. The person can conduct research as many times as she likes, but once she develops an idea, the game ends (she has finished her dissertation).

In this environment, as long as the research and development costs are not too large relative to the potential benefits, TCs will conduct research until they hit upon a sufficiently good idea, and then they (immediately) develop it — specifically, there will be a cutoff \( \bar{v}^* \) such that TCs conduct research until they draw a benefit \( v \geq \bar{v}^* \).

How do naifs behave? If research is more onerous than development — if \( c > k \) — then, as in our basic model, naifs are prone not to start. Hence, in this case naifs might exhibit classic procrastination in that they never get started even when they should; but if they ever do get started, they will eventually develop some idea. It is when development is more onerous than research — when \( k > c \) — that naifs might procrastinate development and thus incur costs without benefits, again much as in our basic model. Moreover, this procrastination might occur after extensive search. In particular, because they are not at all worried about procrastination on development, naifs — like TCs — may be willing to exert a lot of effort to find an idea with large benefits.

To illustrate, suppose \( c = 20, k = 100 \), and \( v \) is distributed uniformly on \([0, 1]\). TCs with \( \delta = .9999 \) conduct research until they find an idea with \( v \geq 0.935 \) and then immediately develop that idea; naifs with \( \delta = .9999 \) and \( \beta = .99 \) conduct research until they find an idea with \( v \geq 0.936 \) but then never develop that idea. In this example, it is well worth finding an idea with high benefits, and so TCs and naifs both search extensively — they both sample, on average, between 15 and 16 projects. But while TCs immediately finish once they find a satisfactory project, naifs procrastinate finishing.

Our second extension is a more striking demonstration of people incurring costs without ever receiving benefits. For many long-term projects, if a person starts a project but then delays before
finishing, she must repeat some of her initial efforts. If, for instance, she works out preliminary results on a research project but then procrastinates in writing the paper, after a while she will not be able to write the paper without reviewing (and perhaps re-generating) her earlier analysis. In such environments, naive procrastination might lead people to repeatedly work out the same preliminary results without ever writing the paper.

Again, we can capture such situations with a simple extension of our model. Specifically, to introduce decay of earlier efforts in a particularly stark manner, suppose that if the person does not finish the project immediately after starting, then she will have to completely re-do the first stage. To focus on procrastination, we once again examine behavior when \( \delta \to 1 \).

Decay of earlier efforts is irrelevant for TCs, and, given \( \delta \to 1 \), TCs clearly start and then finish the project immediately. How does decay of earlier efforts affect naifs? First note that the possibility of decay is irrelevant to the decision whether to start, because, when making this decision, naifs never expect to delay. Second, the possibility of decay makes it less likely that naifs procrastinate finishing. Without decay, waiting merely means delaying finishing by one period. With decay, in contrast, waiting means delaying finishing by two periods and, importantly, having to incur the stage-1 cost for a second time. Hence, the costs of delay are larger with decay, making naifs more motivated to finish. But third, when this extra motivation is not enough to prevent procrastination at stage 2, naifs suffer a particularly unfortunate outcome: They repeatedly start the project but delay finishing, and hence incur the stage-1 cost over and over again.\(^{23}\)

To illustrate, suppose \( c = 3 \), \( k = 50 \), and \( v = 1 \). One can show that, for \( \delta \to 1 \), whereas TCs start and immediately finish this project, naifs with \( \beta = .9 \) repeatedly start but don’t finish. Hence, in an attempt to secure a per-period benefit of 1, naifs end up incurring a cost of 3 every other period without ever receiving any benefits.

5. Discussion and Conclusion

Our analysis in this paper identifies a number of results about naive procrastination on long-term projects. Although our formal analysis focuses on a highly stylized environment, we are confident that most of our conclusions apply qualitatively to more general settings. For instance, while our model focuses on two-stage projects, the lessons can be readily extended to longer-term projects.\(^{23}\) Formally, one can show that naifs never start when \( c \geq \frac{\beta v}{1-\beta} \), naifs start and then finish when \( c < \frac{\beta v}{1-\beta} \) and \( k < \frac{\beta(2v+c)}{1-\beta} \), and naifs repeatedly start the project but never finish when \( c < \frac{\beta v}{1-\beta} \) and \( k \geq \frac{\beta(2v+c)}{1-\beta} \).
The key intuition that drives many of our results is that a person is most prone to procrastinate on the highest-cost stage, and this intuition clearly generalizes. Hence, for many-stage projects, if the highest-cost stage comes first, naive people will either complete the project or never start, whereas if the highest-cost stage occurs later, they might start the project but never finish. Indeed, if the highest-cost stage comes last, naive people might complete every stage of a many-stage project except the last stage, and as a result may expend nearly all of the total cost required to complete the project without receiving benefits.\textsuperscript{24}

While our model assumes that no benefits accrue until after the entire project is completed, our results would also hold qualitatively if the person gets partial benefits from partial completion. Moreover, fixing the total benefit, if some of this benefit accrues upon partial completion, the person is, in fact, \textit{less likely} to complete the project. Intuitively, allocating more benefit following completion of stage 1 and less benefit following completion of stage 2 is much like allocating less cost to stage 1 and more cost to stage 2. Hence, if the person gets partial benefits from partial completion, then she is more prone to start but not finish the project, although the cost of doing so is smaller.

Although we have limited our analysis to the extreme cases of complete sophistication and complete naivete, all of our conclusions apply for the more realistic case of partial naivete. Like sophisticates, partial naifs might not start the project because it is not worthwhile, or because they expect not to want to finish, or because they expect too much delay between starting and finishing. But once again, we doubt the importance of such misbehavior — and it disappears when $\delta \to 1$. More importantly, like naifs, partial naifs might procrastinate on either stage, and they exhibit virtually the same patterns of procrastination.\textsuperscript{25}

The results in this paper highlight a more general theme: the microstructure — or fine details — of environments are important for people with time-inconsistent preferences in ways that don’t matter for people with standard time-consistent preferences. Time-inconsistent people react to the same long-run incentives that time-consistent people react to — \textit{e.g.}, \textit{ceteris paribus}, the higher the benefits and the lower the total effort costs, the more likely are naive procrastinators to complete a project quickly. But time-inconsistent people also react to other, short-run details. Our analysis in Section 3 on endogenizing the microstructure (the cost structure) extends this theme by illustrating that we should not necessarily expect people to choose the best microstructure.

\textsuperscript{24} As an extreme example, if a project involves 1000 days of effort $c = 8$ and just one day of effort $c = 16$, a naive person might put in 1000 days of “low” effort and yet never finish the project.

\textsuperscript{25} Again, for a formal analysis of partial naivete in this environment, see O’Donoghue and Rabin (2002).
Finally, we note that the results in this paper could, if fleshed out, have implications in designing incentives to combat procrastination — both from a managerial perspective and from a government-policy perspective. In O’Donoghue and Rabin (1999b), we explore a simple model of designing a reward scheme to combat procrastination. Our analysis in the present paper suggests not just the importance of designing reward schemes, but also — to the extent possible — of designing projects themselves. For instance, because our analysis in Section 3 shows the potential drawbacks of giving people too much flexibility in how they pursue a project, it suggests that a firm might want to demand a particular schedule of work on a project. Taking our simple model literally, for instance, would suggest that even if there were variation among employees in their disutility of different parts of a project, it might help to impose virtually any schedule on employees rather than to leave it to each employee’s own discretion.
Appendix A: Proofs

Recall that we assume for simplicity that when a person is indifferent between \(a = 0\) and \(a = 1\), she chooses \(a = 1\) (see Footnote 12). Also, we use PPS for perception-perfect strategy.

**Proof of Lemma 1.** We derive conditions for \(s^{TC}\), from which both parts will follow.

Consider stage-2 behavior. Note that, if \(-k + \frac{6v_1}{1-\delta}\geq \delta d \left(-k + \frac{6v_1}{1-\delta}\right)\) for any \(d \in \{1,2,\ldots\}\), then \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t \neq \emptyset\). Because \(-k + \frac{6v_1}{1-\delta}\geq 0\) implies \(-k + \frac{6v_1}{1-\delta}\geq \delta d \left(-k + \frac{6v_1}{1-\delta}\right)\) for any \(d \in \{1,2,\ldots\}\), \(-k + \frac{6v_1}{1-\delta}\geq 0\) implies \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t \neq \emptyset\). Using an analogous argument, \(-k + \frac{6v_1}{1-\delta}< 0\) implies \(s^{TC}(h^t,t) = 0\) for all \(t\) and \(h^t \neq \emptyset\).

Next consider stage-1 behavior. Suppose \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\geq 0\), which implies \(-k + \frac{6v_1}{1-\delta}> 0\) and therefore \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t \neq \emptyset\). Because in this case \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\geq \delta d \left(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\right)\) for any \(d \in \{1,2,\ldots\}\), \(s^{TC}(\emptyset,t) = 1\) for all \(t\). Suppose instead that \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)< 0\). If \(-k + \frac{6v_1}{1-\delta}< 0\) and therefore \(s^{TC}(h^t,t) = 0\) for all \(t\) and \(h^t \neq \emptyset\), then clearly \(s^{TC}(\emptyset,t) = 0\) for all \(t\). And even if \(-k + \frac{6v_1}{1-\delta}> 0\) and therefore \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t \neq \emptyset\), because in this case \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\geq \delta d \left(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\right)\) again \(s^{TC}(\emptyset,t) = 0\) for all \(t\). Summarizing, there are three cases which together imply Lemma 1:

(i) \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)\geq 0\) implies \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t\).

(ii) \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)< 0\) and \(-k + \frac{6v_1}{1-\delta}< 0\) implies \(s^{TC}(h^t,t) = 0\) for all \(t\) and \(h^t\).

(iii) \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)< 0\) and \(-k + \frac{6v_1}{1-\delta}\geq 0\) implies \(s^{TC}(\emptyset,t) = 0\) for all \(t\) and \(s^{TC}(h^t,t) = 1\) for all \(t\) and \(h^t \neq \emptyset\).

**Proof of Lemma 2.** (1) To prove existence, we construct a PPS for all contingencies.

Suppose \(-k + \frac{6v_1}{1-\delta}< 0\). Because \(-k + \frac{6v_1}{1-\delta}< 0\) implies \(-k + \frac{6v_1}{1-\delta}< \beta \delta d \left(-k + \frac{6v_1}{1-\delta}\right)\) for any \(d \in \{1,2,\ldots\}\), any PPS must have \(s^{S}(h^t,t) = 0\) for all \(t\) and \(h^t \neq \emptyset\). Given this, clearly any PPS must have \(s^{S}(\emptyset,t) = 0\) for all \(t\). It follows that, when \(-k + \frac{6v_1}{1-\delta}< 0\), \(s^{S}(h^t,t) = 0\) for all \(t\) and \(h^t\) is a PPS (and it’s unique).

Suppose \(-k + \frac{6v_1}{1-\delta}> 0\). By the definition of \(\gamma\), for any \(s\) and \(h^t \neq \emptyset\), \(\arg \max_{a \in \{0,1\}} V^t(a;h^t,s,\beta) = 0\) if and only if \(\min\{x \in \{1,2,\ldots\} \mid s(h^t,t+x) = 1\} \leq \gamma\). It follows that any PPS must satisfy the following condition: For every \(h^t \neq \emptyset\) there exists \(y \in \{1,\ldots,\gamma+1\}\) such that \(s^{S}(h^t,h^t+x) = 1\) if and only if \(x \in \{y,y+(\gamma+1),y+2(\gamma+1),\ldots\}\). In particular, it is always consistent with PPS to have \(s^{S}(h^t,h^t+x) = 1\) if and only if \(x \in \{1,1+(\gamma+1),1+2(\gamma+1),\ldots\}\). It is then straightforward to show that:

When \(-k + \frac{6v_1}{1-\delta}> 0\) and \(-c + \delta \left(-k + \frac{6v_1}{1-\delta}\right)< 0\), it is a PPS to have \(s^{S}(\emptyset,t) = 0\) for all \(t\) and \(s^{S}(h^t,h^t+x) = 1\) if and only if \(x \in \{1,1+(\gamma+1),1+2(\gamma+1),\ldots\}\).
When \(-k + \frac{\beta \delta v}{1 - \delta} > 0\) and \(-c + \beta \delta \left(-k + \frac{\delta v}{1 - \delta}\right) > 0\), it is a PPS to have \(s^S(\emptyset, t) = 1\) if and only if \(t \in \{1, 1+(z+1), 1+2(z+1), \ldots\}\) and \(s^S(h^t, h^t + x) = 1\) if and only if \(x \in \{1, 1+(\gamma+1), 1+2(\gamma+1), \ldots\}\), where

\[
z \equiv \min \left\{ d \in \{0, 1, \ldots\} \mid -c + \beta \delta \left(-k + \frac{\delta v}{1 - \delta}\right) \geq \beta \delta^{d+1} \left[-c + \delta \left(-k + \frac{\delta v}{1 - \delta}\right)\right]\right\}.
\]

(2) The only other possibility is that they start but never finish, so we must merely rule this out. From above, they never finish only if \(-k + \frac{\beta \delta v}{1 - \delta} < 0\), but in this case they also never start.

(2a) From above, if \(-k + \frac{\beta \delta v}{1 - \delta} < 0\) then the unique PPS involves never starting. Suppose instead that \(-k + \frac{\beta \delta v}{1 - \delta} > 0\) and \(-c + \beta \delta \left(-k + \frac{\delta v}{1 - \delta}\right) < 0\), which together imply \(-c + \beta \delta^{d+1} \left(-k + \frac{\delta v}{1 - \delta}\right) < 0\) for any \(d \in \{1, 2, \ldots\}\). In this case, in every period the person prefers never starting over starting now, and as a result any PPS must involve never starting.

(2b) Suppose there exists a PPS \(s^S\) that involves never starting. Then for any \(t, a = 0\) yields \(V^t(a; \emptyset, s^S, \beta) = 0\), while \(a = 1\) yields \(V^t(a; \emptyset, s^S, \beta) \geq -c + \beta \delta^{\gamma+1} \left(-k + \frac{\delta v}{1 - \delta}\right)\). Hence, \(-c + \beta \delta^{\gamma+1} \left(-k + \frac{\delta v}{1 - \delta}\right) > 0\) contradicts that \(s^S(\emptyset, t) = 0\). It follows that every PPS must involve completing the project.

(2c) The proof of Part 1 constructs a PPS that involves completing the project. Hence, it remains to construct a PPS that involves never starting. Consider a strategy \(s\) that has \(s(\emptyset, t) = 0\) for all \(t\) and \(s(h^t, h^t + x) = 1\) if and only if \(x \in \{\gamma+1, 2(\gamma+1), 3(\gamma+1), \ldots\}\). From the proof of part 1, period-2 behavior is consistent with PPS. Since for any \(t, a = 0\) yields \(V^t(a; \emptyset, s, \beta) = 0\), while \(a = 1\) yields \(V^t(a; \emptyset, s, \beta) = -c + \beta \delta^{\gamma+1} \left(-k + \frac{\delta v}{1 - \delta}\right) < 0\), \(s(\emptyset, t) = 0\) is also consistent with PPS.

**Proof of Proposition 1.** From Lemma 1, TCs never start if and only if \(-c - \delta k + \frac{\delta^2 v}{1 - \delta} = -C + V < 0\), or \(C/V > 1\). From Lemma 2, sophisticates never start only if either (i) \(-k + \frac{\beta \delta v}{1 - \delta} < 0\), (ii) \(-c - \beta \delta k + \frac{\beta \delta^2 v}{1 - \delta} < 0\), or (iii) \(-k + \frac{\beta v}{1 - \delta} \geq 0\) and \(-c + \beta \delta^{\gamma+1} \left(-k + \frac{\delta v}{1 - \delta}\right) < 0\). For case (i), \(-k + \frac{\beta \delta v}{1 - \delta} < 0\) implies \(0 > -\delta k + \frac{\delta^2 v}{1 - \delta} > -c - \delta k + \frac{\delta^2 v}{1 - \delta} = -C + \beta V\), and so \(C/V > \beta\). For case (ii), \(0 > -c - \beta \delta k + \frac{\beta \delta^2 v}{1 - \delta} = -C + \beta V + (1 - \beta) \delta k > -C + \beta V\) (given \(k > 0\)), and so \(C/V > \beta\). For case (iii), by the definition of \(\gamma, \beta \delta^\gamma \left(-k + \frac{\delta v}{1 - \delta}\right) > -k + \frac{\beta v}{1 - \delta} > 0\), which implies \(0 > -c + \beta \delta^{\gamma+1} \left(-k + \frac{\delta v}{1 - \delta}\right) > -c + \delta \left(-k + \frac{\delta v}{1 - \delta}\right) = -C + \beta V\), and so \(C/V > \beta\).

**Proof of Lemma 3.** (1) Since there is a unique \(s^{TC}\), and since, for all \(t\) and \(h^t\), any \(s^N\) must specify an optimal action conditional on following that \(s^{TC}\) in the future, it follows that \(s^N\) is unique (given that we assume people choose \(a = 1\) when indifferent).

(2) Consider stage-1 behavior. If \(-c - \delta k + \frac{\delta^2 v}{1 - \delta} < 0\), naifs most prefer never starting, and so \(s^N(\emptyset, t) = 0\) for all \(t\). Suppose \(-c - \delta k + \frac{\delta^2 v}{1 - \delta} \geq 0\) and thus \(s^{TC}(h^t, t) = 1\) for all \(t\) and \(h^t\). For any \(t\) and \(h^t = \emptyset\), naifs compare completion beginning now to completion beginning next period, and
they prefer the former when \(-c + \beta \delta \left( -k + \frac{\delta v}{1-\delta} \right) \geq \beta \delta \left( -c - \delta k + \frac{\delta^2 u}{1-\delta} \right)\).

Consider stage-2 behavior. If \(-k + \frac{\delta v}{1-\delta} < 0\), naifs most prefer never finishing, and so \(s^N(h^t, t) = 0\) for all \(t\) and \(h^t \neq \emptyset\). Suppose \(-k + \frac{\delta v}{1-\delta} \geq 0\) and thus \(s^{TC}(h^t, t) = 1\) for all \(t\) and \(h^t \neq \emptyset\). For any \(t\)
and \(h^t \neq \emptyset\), naifs compare finishing now to finishing next period, and they prefer the former when
\(-k + \frac{\beta \delta v}{1-\delta} \geq \beta \delta \left( -k + \frac{\delta v}{1-\delta} \right)\).

Finally, it is straightforward to show that the condition for completing stage 1 does not imply the condition for completing stage 2, and hence naifs might start but not finish.

**Proof of Proposition 2.** For each case, we prove there exists \(\bar{\delta} < 1\) such that the statement holds for all \(\delta \in (\bar{\delta}, 1)\).

1. TCs complete the project if \(-c - \delta k + \frac{\delta^2 v}{1-\delta} \geq 0\). Because \(\lim_{\delta \to 1} \left[ -c - \delta k + \frac{\delta^2 v}{1-\delta} \right] = \infty\), there exists \(\bar{\delta} < 1\) such that TCs complete the project for all \(\delta \in (\bar{\delta}, 1)\). For sophisticates, any PPS involves completing the project if both \(-k + \frac{\beta \delta v}{1-\delta} > 0\) and \(-c + \beta \delta^\gamma + 1 \left( -k + \frac{\delta v}{1-\delta} \right) > 0\). With a little work, one can prove that \(\lim_{\delta \to 1} \gamma\) is finite. Because for any \(d\), \(\lim_{\delta \to 1} \left[ -c + \beta \delta^{d+1} \left( -k + \frac{\delta v}{1-\delta} \right) \right] = \infty\), and because \(\lim_{\delta \to 1} \left[ -k + \frac{\beta \delta v}{1-\delta} \right] = \infty\), it follows that there exists \(\bar{\delta} < 1\) such that for all \(\delta \in (\bar{\delta}, 1)\) any PPS for sophisticates involves completing the project.

2. Naifs complete stage 1 when \(-c + \beta \delta \left( -k + \frac{\delta v}{1-\delta} \right) \geq \beta \delta \left( -c - \delta k + \frac{\delta^2 v}{1-\delta} \right)\), which we can rewrite as \(c \leq -\frac{\beta \delta (1-\delta) k}{1-\beta} + \frac{\beta \delta^2 v}{1-\beta}\). If \(c \geq \frac{\beta v}{1-\beta}\), then since \(\frac{\beta v}{1-\beta} > \frac{\beta \delta^2 v}{1-\beta}\) for all \(\delta < 1\), it follows that \(c > -\frac{\beta \delta (1-\delta) k}{1-\beta} + \frac{\beta \delta^2 v}{1-\beta}\) for all \(\delta < 1\), and therefore for all \(\delta < 1\) naifs do not complete stage 1. Suppose \(c < \frac{\beta v}{1-\beta}\). Because \(\lim_{\delta \to 1} \left[ -\frac{\beta \delta (1-\delta) k}{1-\beta} + \frac{\beta \delta^2 v}{1-\beta} \right] = \frac{\beta v}{1-\beta}\), there exists \(\bar{\delta} < 1\) such that \(c < -\frac{\beta \delta (1-\delta) k}{1-\beta} + \frac{\beta \delta^2 v}{1-\beta}\) for all \(\delta \in (\bar{\delta}, 1)\), and therefore naifs complete stage 1 for all \(\delta \in (\bar{\delta}, 1)\). The argument for stage 2 is analogous.

**Proof of Proposition 3.** Naifs complete stage 1 if \(-c + \beta \delta \left( -k + \frac{\delta v}{1-\delta} \right) \geq \beta \delta \left( -c - \delta k + \frac{\delta^2 v}{1-\delta} \right)\), which we can rewrite as \(k \leq -\frac{1-\beta \delta}{\beta \delta (1-\delta)} + \frac{\delta v}{1-\delta}\). Naifs do not complete stage 2 if \(k > \frac{\beta v}{1-\beta}\). We need to prove that for all \(\beta, \delta, c\), there exists \(k\) and \(v\) such that both conditions are satisfied. Because \(\frac{k}{\beta v} > \frac{\beta}{\beta (1-\delta)}\), there exists \(v'\) such that for all \(v > v'\), \(-\frac{(1-\beta \delta) c}{\beta \delta (1-\delta)} + \frac{\delta v}{1-\delta} > \frac{\beta v}{1-\beta}\). For any such \(v\), both conditions are satisfied for any \(\bar{k} \in \left( \frac{\beta \delta v}{1-\beta}, -\frac{(1-\beta \delta) c}{\beta \delta (1-\delta)} + \frac{\delta v}{1-\delta} \right)\).

**Proof of Lemma 4.** Because every \((q, r) \in P(A, \bar{a})\) has \(q + r = A\), we transform the problem into choosing an \(r \in [A - \bar{a}, \bar{a}]\).

1. Because TCs pursue an optimal path, if they do any project, it will be the project that maximizes \(g(r) \equiv - (A - r) - \delta r + \frac{\delta^2 u}{1-\delta}\). Because \(\frac{dg}{dr} = 1 - \delta > 0\), the best project has \(r = \bar{a}\). Finally, they prefer doing project \((A - \bar{a}, \bar{a})\) to never starting if and only if \(-(A - \bar{a}) - \delta \bar{a} + \frac{\delta^2 u}{1-\delta} \geq 0\).

2. Because naifs plan to pursue an optimal path, if they start any project, it will be the
project that maximizes \( \hat{g}(r) \equiv -(A - r) - \beta \delta r + \frac{\beta \delta^2 r}{v r} \), and since \( \frac{dv}{dr} = 1 - \beta \delta > 0 \), the best project has \( r = \bar{a} \). But much as in Lemma 3, naifs start project \((A - \bar{a}, \bar{a})\) if and only if they prefer completing the project beginning now rather than beginning next period, which holds when \(- (A - \bar{a}) + \beta \delta \left( - \bar{a} + \frac{\delta v}{1 - \beta} \right) \geq \beta \delta \left[ - (A - \bar{a}) - \delta \bar{a} + \frac{\delta^2 v}{1 - \beta} \right] \). Finally, if naifs complete stage 1, the condition for whether they complete stage 2 is identical to the condition in Lemma 3 when \( k = \bar{a} \).

(3) Using the logic from Lemma 2, for any project \((A - r, r) \in \mathbf{P}(A, \bar{a})\), it is consistent with PPS to complete stage 2 immediately after completing stage 1 if and only if \(- r + \frac{\beta \delta v}{1 - \beta} \geq 0\). Moreover, for any project \((A - r, r)\) with \(- r + \frac{\beta \delta v}{1 - \beta} < 0\), any PPS must involve never finishing that project.

Given this stage-2 behavior, consider stage-1 behavior. If sophisticates start any project, it will be the project that maximizes \( \hat{g}(r) \) such that \( r \leq \frac{\beta \delta v}{1 - \beta} \) (they’ll never start a project that they expect not to finish). It follows that the best project has \( r = r_o \). If \(- (A - r_o) + \beta \delta \left( - r_o + \frac{\delta v}{1 - \beta} \right) > 0\), completion of project \((A - r_o, r_o)\) beginning now is preferred to never starting, which implies they must eventually start. It follows that if \(- (A - r_o) + \beta \delta \left( - r_o + \frac{\delta v}{1 - \beta} \right) > 0\) then there exists a PPS under which sophisticates complete project \((A - r_o, r_o)\). (Note: \(- (A - r_o) + \beta \delta \left( - r_o + \frac{\delta v}{1 - \beta} \right) > 0\) implies \( r_o > A - \bar{a} \).)

Finally, suppose \(- (A - r_o) + \beta \delta \left( - r_o + \frac{\delta v}{1 - \beta} \right) < 0\). If \( \frac{\beta \delta v}{1 - \beta} < A - \bar{a} \), then there does not exist any \( r \in [A - \bar{a}, \bar{a}] \) such that they’d finish, and so clearly they never start any project. If \( \frac{\beta \delta v}{1 - \beta} \geq A - \bar{a} \), then for any project that they’d finish, sophisticates always prefer never starting to starting now, and so they never start any project.

**Proof of Proposition 4.** (1) The logic is identical to that for Proposition 2 (for sophisticates, note that there exists \( \delta' < 1 \) such that \( r_o = \bar{a} \) for all \( \delta \in (\delta', 1) \)).

(2) Combining Lemma 4 and Proposition 2, under endogenous cost structure \( \mathbf{P}(c + k, \bar{a}) \), naifs complete stage 1 if and only if \((c + k) - \bar{a} < \frac{\beta \delta}{1 - \beta} \), and they complete stage 2 if and only if \( \bar{a} < \frac{\beta v}{1 - \beta} \). And from Proposition 2, under exogenous cost structure \((c, k)\), naifs complete stage 1 if and only if \( c < \frac{\beta v}{1 - \beta} \), and they complete stage 2 if and only if \( k < \frac{\beta v}{1 - \beta} \). Because \((c + k) - \bar{a} \leq c \) (given \( k \leq \bar{a} \)), they are more likely to complete stage 1 under \( \mathbf{P}(c + k, \bar{a}) \) than under \((c, k)\); and because \( \bar{a} \geq k \), they are less likely to complete stage 2 under \( \mathbf{P}(c + k, \bar{a}) \) than under \((c, k)\).
References


